Summer Review Assignments for AP® Calculus AB (12th graders)

Dear Parents and Students:

Your teachers for next year are already making plans for our new school year, and we are all looking forward to seeing you in August.

These summer assignments are designed to be a review of the math skills expected of a student at Lanier Christian Academy entering the AP® Calculus AB course. These math skills are very important to the success of your child this year. The summer review work will prepare students for the assessment, and allow teachers more instructional time and the ability to progress into new material sooner.

· Print the assignment.

· Summer work is to be turned in on the first Monday of the first school week.

· Students must show their work and put a box around the answer.

· Parents, check the answers and mark incorrect answers with a colored pen, pencil, or marker. The answers are included. Write the grade in the UPPER RIGHT corner (number correct divided by number of assigned problems times 100). Also, please sign your name to indicate that the work was checked by you. If your student misses 20% or more of the problems, your student needs to correct the missed problems to the right of the original work or on a separate piece of notebook paper. Staple all pages together. Your student should make corrections until the grade is AT LEAST an 80.

***Points will be deducted if students do not show all their work & corrections, if parents do not grade and sign the work, and if the work turned in is below 80% correct.

Thanks for working on this review and have a wonderful summer!

**** Feel free to ask question via Groupme!

Lanier Christian Academy **Name** ______________________________

AP Calculus AB

Summer Review Packet **Score** ______________________ **Period** ______

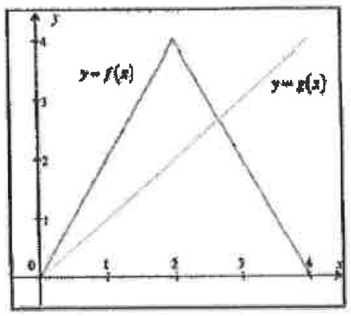
Topic A: Functions

1.) If $f(x) = 4x - x^2$, find:

a.) $\sqrt{f\left(\frac{3}{2}\right)}$ b.) $\frac{f(x+h)-f(x)}{h}$

2.) If $V(r) = \frac{4}{3}\pi r^3$, find: $V(r+1) - V(r-1)$

3.) If $f(x)$ and $g(x)$ are given in the graph, find:



a.) $(f-g)(3)$ b.) $f(g(3))$

4.) If $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \le x < 2 \\ \sqrt{x+2} - 2, & x \ge 2 \end{cases}$, find:

a.) $f(0) - f(2)$ b.) $f(f(3))$

Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.) $y = x^3 - x^2 + x$

2.) $y = \dfrac{x-4}{x^2-16}$

3.) $y = \sqrt{2x-9}$

4.) $y = \log(x-10)$

5.) $y = \dfrac{\sqrt{2x+14}}{x^2-49}$

Find the range of the following functions:

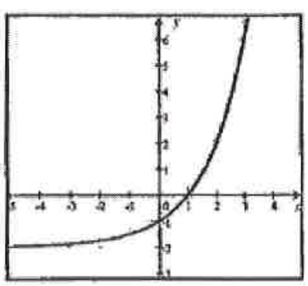
9.) $y = x^4 + x^2 - 1$

10.) $y = 100^x$

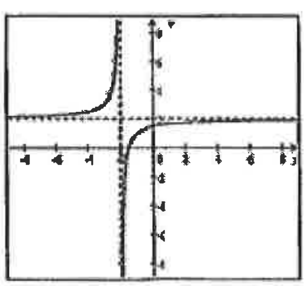
11.) $y = \sqrt{x^2+1} + 1$

Find the domain and range of the following functions using interval notation.

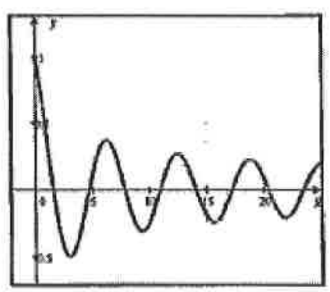
12.)



13.)



14.)



Topic D: Even/Odd Functions and Symmetry

Show work to determine if the relation is even, odd, or neither. You may want to research how to determine evenness and oddness.

1.) $f(x)=7$

2.) $f(x)=2x^2-4x$

3.) $f(x)=-3x^3-2x$

4.) $f(x)=\sqrt{x+1}$

5.) $f(x)=\sqrt{x^2+1}$

6.) $f(x)=|8x|$

Topic E: Function Transformations

If $f(x)=x^2-1$, describe in words what the following would do to the graph of $f(x)$:

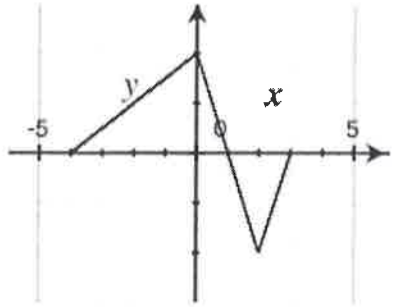
1.) $-f(x+2)$

2.) $5f(x)+3$

3.) $f(2x)$

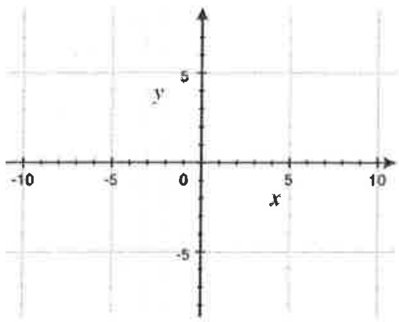
4.) $|f(x)|$

Here is a graph of $y = f(x)$:

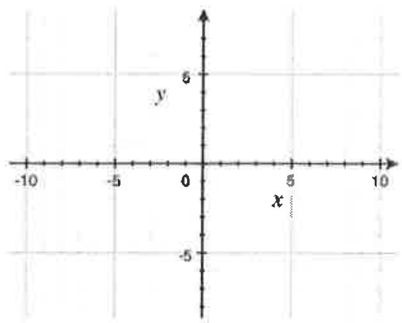


Sketch the following graphs:

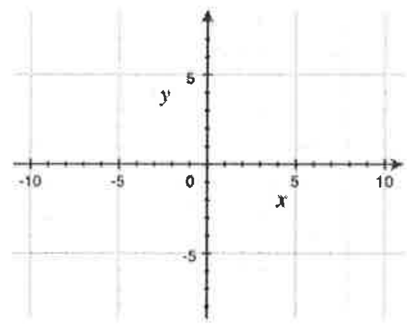
7.) $y = 2f(x)$



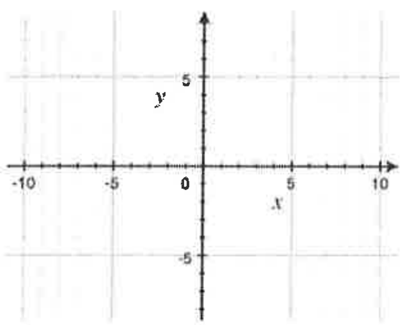
8.) $y = -f(x)$



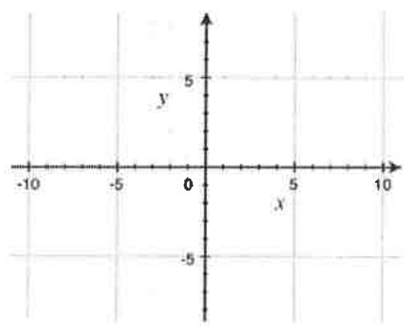
9.) $y = f(x-1)$



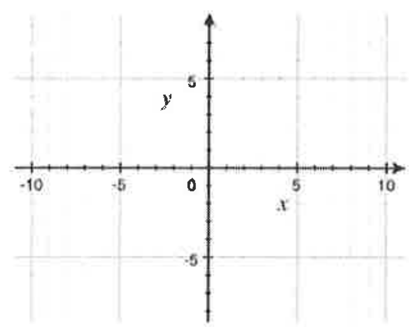
10.) $y = f(x) + 2$



11.) $y = |f(x)|$



12.) $y = f(|x|)$



Topic F: Special Factorization

Factor completely.

1.) $27x^3 - 125y^3$ 2.) $x^4 + 11x^2 - 80$ 3.) $ac + cd - ab - bd$ 4.)

$2x^2 + 50y^2 - 20xy$

7.) $x^2 + 12x + 36 - 9y^2$

8.) $x^3 - xy^2 + x^2y - y^3$

9.) $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Topic G: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m = \frac{2}{3}, \left(-6, \frac{1}{3}\right)$

2.) Find the equation of the line in point-slope form, passing through the given points

a.) $\left(-2, \frac{2}{3}\right), \left(\frac{1}{2}, 1\right)$

3.) Find the equation of the line in general form, containing the point $(4, -2)$ and parallel to the line containing the points $(-1, 4)$ and $(2, 3)$.

Topic H: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

1.) $x^2 + x + \frac{1}{4} = 0$

2.) $2x^2 - 72 = 0$

3.) $81x^2 + 72x + 16 = 0$

4.) $x + \frac{1}{x} = \frac{17}{4}$

5.) $x^3 - 5x^2 + 5x - 25 = 0$

Topic I: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) $y = \frac{2x^2 + 6x}{x^2 + 5x + 6}$

2.) $y = \frac{x}{x^2 - 25}$

3.) $y = \frac{x^3}{x^2 + 4}$

4.) $y = \frac{x^3 + 4x}{x^3 - 2x^2 + 4x - 8}$

5.) $y = \frac{10x + 20}{x^3 - 2x^2 - 4x + 8}$

Topic J: Negative and Fractional Exponents

Simplify and write with positive exponents.

1.) $-12^2 x^{-5}$

2.) $\left(4x^{-1}\right)^{-1}$

3.) $\left(\frac{5x^3}{y^2}\right)^{-3}$

4.) $\left(x^3-1\right)^{-2}$

5.) $\left(-32x^{-5}\right)^{-3/5}$

6.) $\frac{1}{4}\left(16x^2\right)^{-3/4}(32x)$

7.) $\frac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$

8.) $\left(x^{-2}+2^{-2}\right)^{-1}$

Topic K: Complex Fractions

Eliminate the complex fractions:

1.) $\frac{x^{-1}+y^{-1}}{x+y}$

2.) $\frac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

3.) $\frac{\frac{1}{3}(3x-4)^{-3/4}}{-\frac{3}{4}}$

4.) $\frac{2x(2x-1)^{1/2}-2x^2(2x-1)^{-1/2}}{(2x-1)}$

Topic L: Inverses

Find the inverse of each of the following functions.

1.) $y = ax + b$ 2.) $y = 9 - x^2,\ x \geq 0$ 3.) $y = \sqrt{1 - x^3}$ 4.) $y = \dfrac{2x+1}{3-2x}$

Find the inverse of each of the following functions and show that $f\left(f^{-1}(x)\right) = x$

5.) $f(x) = \dfrac{1}{2}x - \dfrac{4}{5}$ 6.) $f(x) = \dfrac{x^2}{x^2 + 1}$

10.) Without finding the inverse, find the domain and range of the inverse to $f(x) = \dfrac{\sqrt{x+1}}{x^2}$

Topic M: Adding Fractions and Solving Rational Equations

1.) Combine the following fractions:

a.) $\dfrac{5}{2x}-\dfrac{5}{3x+15}$

b.) $\dfrac{2x-1}{x-1}-\dfrac{3x}{2x+1}$

2.) Solve the equation for x.

a.) $\dfrac{5}{2x}-\dfrac{5}{3(x+5)}=\dfrac{5}{x}$

b.) $\dfrac{2x-1}{x-1}-\dfrac{3x}{2x+1}=\dfrac{x^2+11}{2x^2-x-1}$

Topic N: Absolute Value Equations

Solve the following equations:

1.) $|4x-5|+5x+2=0$

2.) $|x^2-2x-1|=7$

Topic O: Solving Inequalities

Solve the following inequalities:

1.) $\frac{3}{4} > x+1 > \frac{1}{2}$

2.) $x+7 \geq |5-3x|$

3.) $(x+2)^2 < 25$

4.) $\frac{5}{x-6} \geq \frac{1}{x+2}$

5.) Find the domain of: $\sqrt{\frac{x^2-x-6}{x-4}}$

Topic P: Exponential Functions and Logarithms

Simplify the following:

1.) $\log_2 \frac{1}{4}$

2.) $\ln \frac{1}{\sqrt[3]{e^2}}$

3.) $5^{\log_5 40}$

4.) $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

5.) $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$

6.) $\log_3 \left(\sqrt{3}\right)^5$

Solve the following:

7.) $\log_9 \left(x^2 - x + 3\right) = \frac{1}{2}$

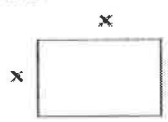
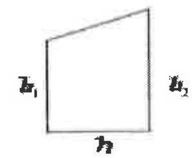
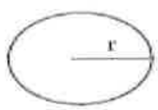
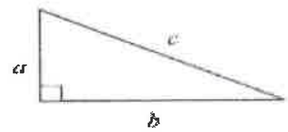
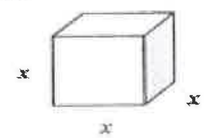
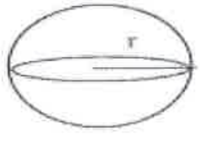
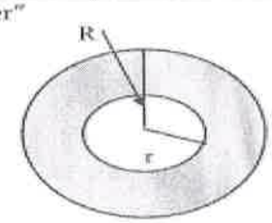
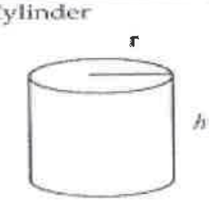
8.) $\log(x-3) + \log 5 = 2$

9.) $\log_5 (x+3) - \log_5 x = 2$

10.) $3^{x-2} = 18$ 11.) $e^{3x+1} = 10$ 12.) $8^x = 5^{2x-1}$

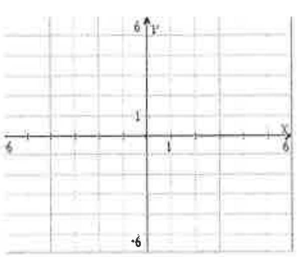
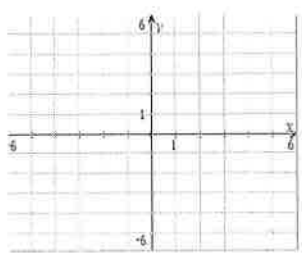
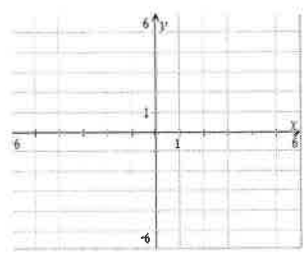
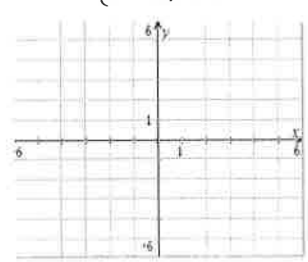
Topic Q: Geometry

1.) You will use each of the following formulas in AP Calculus AB. Complete each of the following.

Square  Perimeter = __________ Area = __________	Rectangle  Perimeter = __________ Area = __________	Trapezoid  Area = __________
Circle  Circumference = __________ Area = __________	Triangle  Pythagorean Theorem (only good for right triangles) = __________ Area (of any triangle) = __________	Cube  Volume = __________ Surface Area = __________
Sphere  Volume = __________	"Washer"  Area of the shaded region = __________	Cylinder  Volume = __________

Find the area between the x-axis and $f(x)$ from $x = 0$ to $x = 5$. Sketch the region to verify.

2.) $f(x) = 4$ 3.) $f(x) = x + 3$ 4.) $f(x) = \sqrt{9 - x^2}$ 5.) $f(x) = \begin{cases} x+1, & x \le 2 \\ 5-x, & x > 2 \end{cases}$

Solutions

Topic A: Functions

1.) If $f(x) = 4x - x^2$, find:

a.) $\sqrt{f\left(\frac{3}{2}\right)}$

$= \sqrt{4\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2}$

$= \sqrt{6 - \frac{9}{4}} = \sqrt{\frac{15}{4}} = \frac{\sqrt{15}}{2}$

$\boxed{\sqrt{f\left(\frac{3}{2}\right)} = \frac{\sqrt{15}}{2}}$

b.) $\frac{f(x+h) - f(x)}{h}$

$= \frac{\left[4(x+h) - (x+h)^2\right] - \left[4x - x^2\right]}{h}$

$= \frac{\left[(4x+4h) - (x^2 + 2xh + h^2)\right] - \left[4x - x^2\right]}{h}$

$= \frac{\left[4h - (2xh + h^2)\right]}{h} = \frac{\left[4h - 2xh - h^2\right]}{h}$

$= \boxed{4 - 2x - h = \frac{f(x+h) - f(x)}{h}}$

2.) If $V(r) = \frac{4}{3}\pi r^3$, find:

$V(r+1) - V(r-1)$

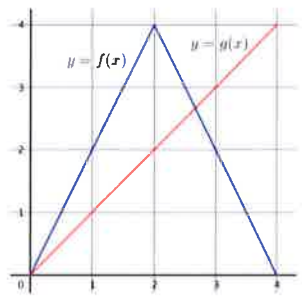
$= \frac{4}{3}\pi(r+1)^3 - \frac{4}{3}\pi(r-1)^3$

$= \frac{4}{3}\pi\left[(r+1)^3 - (r-1)^3\right]$

$= \frac{4}{3}\pi\left[(r^3 + 3r^2 + 3r + 1) - (r^3 - 3r^2 + 3r - 1)\right]$

$= \boxed{\frac{4}{3}\pi(6r^2 + 2) = V(r+1) - V(r-1)}$

3.) If $f(x)$ and $g(x)$ are given in the graph, find:



a.) $(f - g)(3)$

$= f(3) - g(3) = 2 - 3 = \boxed{-1}$

b.) $f(g(3))$

$= f(3) = \boxed{2}$

4.) If $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \le x < 2 \\ \sqrt{x+2} - 2, & x \ge 2 \end{cases}$, find:

a.) $f(0) - f(2)$

$= (0^2 - 1) - \sqrt{2+2}$

$= -1 - 2 = \boxed{-3}$

b.) $f(f(3))$

$f(3) = \sqrt{5} - 2$

$f(\sqrt{5} - 2) =$

$\boxed{(\sqrt{5} - 2)^2 - 1 = 4 - 4\sqrt{5} \approx -4.9442\ldots}$

Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.) $y = x^3 - x^2 + x$

$x \in (-\infty, \infty)$

2.) $y = \dfrac{x-4}{x^2-16}$

$y = \dfrac{x-4}{(x-4)(x+4)} = \dfrac{1}{x+4}, x \neq 4$

$x \in (-\infty, 4) \cup (4, \infty)$

3.) $y = \sqrt{2x-9}$

$2x - 9 \geq 0 \Rightarrow x \geq \dfrac{9}{2}$

$x \in \left(\dfrac{9}{2}, \infty\right)$

4.) $y = \log(x-10)$

$x - 10 > 0 \Rightarrow x > 10$

$x \in (10, \infty)$

5.) $y = \dfrac{\sqrt{2x+14}}{x^2-49}$

$2x + 14 \geq 0 \Rightarrow x \geq -7$

$x^2 - 49 \neq 0 \Rightarrow x = \pm 7$

$x \in (-7, 7) \cup (7, \infty)$

Find the range of the following functions:

9.) $y = x^4 + x^2 - 1$

$x = 0 \Rightarrow y = -1$

$y = x^4 + x^2 - 1 \Rightarrow$

only positive numbrs

are added to -1

$y \in [-1, \infty)$

10.) $y = 100^x$

$y \in (0, \infty)$

11.) $y = \sqrt{x^2+1} + 1$

$x^2 + 1 \geq 1$

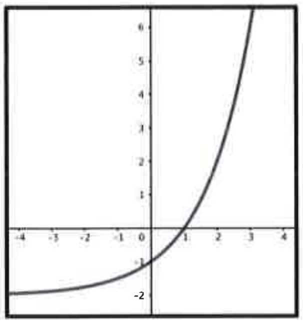
$\sqrt{x^2+1} \geq 1$

$\sqrt{x^2+1} + 1 \geq 2$

$y \in [2, \infty)$

Find the domain and range of the following functions using interval notation.

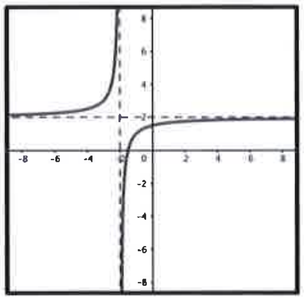
12.)



$x \in (-\infty, \infty)$

$y \in (-2, \infty)$

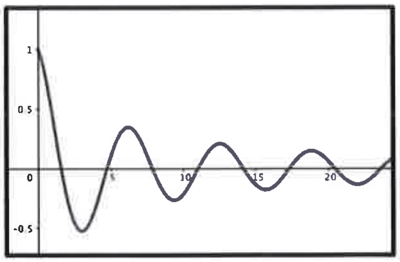
13.)



$x \in (-\infty, -2) \cup (-2, \infty)$

$y \in (-\infty, 2) \cup (2, \infty)$

14.)



$x \in [0, \infty)$

$y \in [-0.5, 1]$

Topic D: Even/Odd Functions and Symmetry

Show work to determine if the relation is even, odd, or neither. You may want to research how to determine evenness and oddness.

1.) $f(x)=7$

for all $x=c$

$f(c)=7 \quad f(-c)=7$

$\left.\begin{matrix} f(c)=f(-c) \\ f(c)\neq -f(-c) \end{matrix}\right\} \Rightarrow \boxed{f \text{ is even}}$

2.) $f(x)=2x^2-4x$

for all $x=c$

$f(c)=2c^2-4c$

$f(-c)=2(-c)^2-4(-c)$

$=2c^2+4c$

$\left.\begin{matrix} f(c)\neq f(-c) \\ f(c)\neq -f(-c) \end{matrix}\right\} \Rightarrow \boxed{f \text{ is neither}}$

3.) $f(x)=-3x^3-2x$

for all $x=c$

$f(c)=-3c^3-2c$

$f(-c)=-3(-c)^3-2(-c)$

$=3c^3+2c$

$\left.\begin{matrix} f(c)\neq f(-c) \\ f(c)=-f(-c) \end{matrix}\right\} \Rightarrow \boxed{f \text{ is odd}}$

4.) $f(x)=\sqrt{x+1}$

for all $x=c$

$f(c)=\sqrt{c+1}$

$f(-c)=\sqrt{-c+1}$

$\left.\begin{matrix} f(c)\neq f(-c) \\ f(c)\neq -f(-c) \end{matrix}\right\} \Rightarrow \boxed{f \text{ is neither}}$

5.) $f(x)=\sqrt{x^2+1}$

for all $x=c$

$f(c)=\sqrt{c^2+1}$

$f(-c)=\sqrt{(-c)^2+1}=\sqrt{c^2+1}$

$\left.\begin{matrix} f(c)=f(-c) \\ f(c)\neq -f(-c) \end{matrix}\right\} \Rightarrow \boxed{f \text{ even}}$

6.) $f(x)=|8x|$

for all $x=c$

$f(c)=|8c|$

$f(-c)=|8(-c)|=|8c|$

$\left.\begin{matrix} f(c)=f(-c) \\ f(c)\neq -f(-c) \end{matrix}\right\} \Rightarrow \boxed{f \text{ even}}$

Topic E: Function Transformations

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1.) $-f(x+2)$

The graph of f is shifted left 2 and reflected over the x–axis.

2.) $5f(x)+3$

The graph of f is vertically stretched by a factor of 5 and shifted up 3.

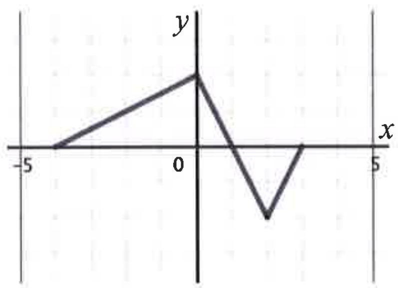
3.) $f(2x)$

The graph of f is horizontally shrunken (compressed) by a factor of 2.

4.) $|f(x)|$

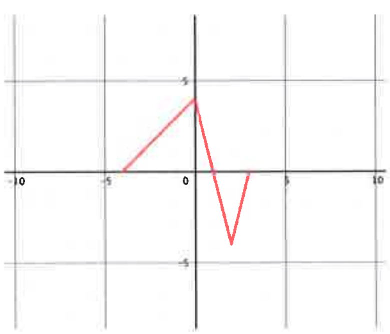
The graph of f has any portion below the x–axis reflected above the x–axis.

Here is a graph of $y = f(x)$:

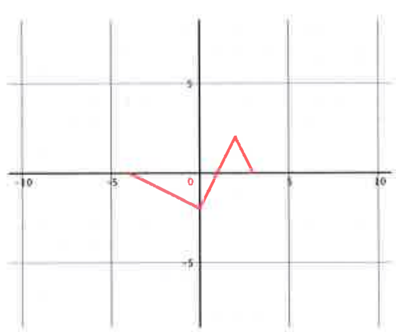


Sketch the following graphs:

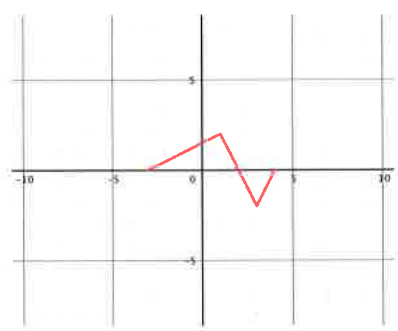
7.) $y = 2f(x)$



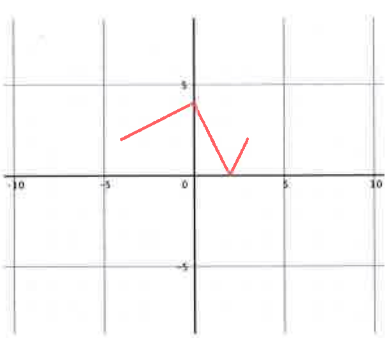
8.) $y = -f(x)$



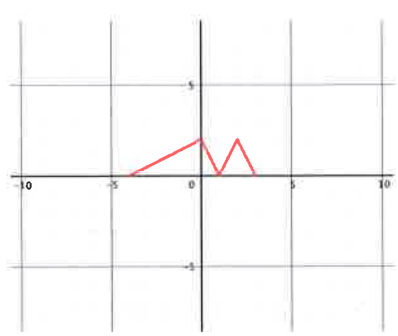
9.) $y = f(x-1)$



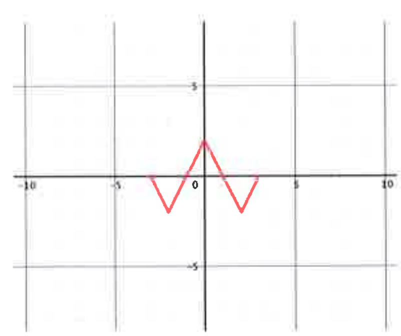
10.) $y = f(x)+2$



11.) $y = |f(x)|$



12.) $y = f(|x|)$



Topic F: Special Factorization

Factor completely.

1.) $27x^3 - 125y^3$

$(3x)^3 - (5y)^3 \Rightarrow (3x - 5y)$ is a factor

$$\boxed{(3x-5y)(9x^2+15xy+25y^2)}$$

$$\begin{array}{r} 9x^2 + 15xy + 25y^2 \\ 3x - 5y \overline{)\, 27x^3 \qquad\qquad - 125y^3} \\ \underline{27x^3 - 45x^2y} \\ 45x^2y \\ \underline{45x^2y - 75xy^2} \\ 75xy^2 - 125y^3 \\ \underline{75xy^2 - 125y^3} \\ 0 \end{array}$$

2.) $x^4 + 11x^2 - 80$

$\underbrace{(x^2-5)}_{\text{difference of 2 squares}}(x^2+16)$

$$\boxed{(x+\sqrt{5})(x-\sqrt{5})(x^2+16)}$$

3.) $ac + cd - ab - bd$

$(ac - ab) + (cd - bd)$

$a(c-b) + d(c-b)$

$$\boxed{(c-b)(a+d)}$$

4.) $2x^2 + 50y^2 - 20xy$

$2\underbrace{(x^2 - 10xy + 25y^2)}_{\text{perfect square}}$

$$\boxed{2(x-5y)^2}$$

7.) $x^2 + 12x + 36 - 9y^2$

$\underbrace{x^2 + 12x + 36}_{\text{perfect square}} - 9y^2$

$\underbrace{(x+6)^2 - 9y^2}_{\text{difference of squares}}$

$((x+6)-3y)((x+6)+3y)$

$$\boxed{(x+6-3y)(x+6+3y)}$$

8.) $x^3 - xy^2 + x^2y - y^3$

$x = y \Rightarrow x^3 - x(x^2) + x^2(x) - x^3 = 0$

$(x-y)$ is a factor

$(x-y)\underbrace{(x^2+2xy+y^2)}_{\text{perfect square}}$

$$\boxed{(x-y)(x+y)^2}$$

$$\begin{array}{r} x^2 + 2xy + y^2 \\ x - y \overline{)\, x^3 + x^2y - xy^2 - y^3} \\ \underline{x^3 - x^2y} \\ 2x^2y - xy^2 \\ \underline{2x^2y - 2xy^2} \\ xy^2 - y^3 \\ \underline{xy^2 - y^3} \\ 0 \end{array}$$

9.) $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

$(x-3)^2(2x+1)^2[(2x+1)+(x-3)]$

$$\boxed{(x-3)^2(2x+1)^2(3x-2)}$$

Topic G: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m=\frac{2}{3}, \left(-6, \frac{1}{3}\right)$

$$y-\left(\frac{1}{3}\right)=\frac{2}{3}(x-(-6))$$

$$\boxed{y=\frac{1}{3}+\frac{2}{3}(x+6)}$$

2.) Find the equation of the line in point-slope form, passing through the given points.

a.) $\left(-2, \frac{2}{3}\right), \left(\frac{1}{2}, 1\right)$

$$m=\frac{1-\frac{2}{3}}{\frac{1}{2}-(-2)}=\frac{1/3}{5/2}=\frac{2}{15}$$

$$y-1=\frac{2}{15}\left(x-\frac{1}{2}\right)$$

$$\boxed{y=1+\frac{2}{15}\left(x-\frac{1}{2}\right)}$$ or

$$y-\frac{2}{3}=\frac{2}{15}(x-(-2))$$

$$\boxed{y=\frac{2}{3}+\frac{2}{15}(x+2)}$$

3.) Find the equation of the line in general form, containing the point $(4,-2)$ and parallel to the line containing the points $(-1,4)$ and $(2,3)$. $m=\frac{3-4}{2-(-1)}=\frac{-1}{3}=-\frac{1}{3}$ $\quad y-(-2)=-\frac{1}{3}(x-4)\Rightarrow 3y-3(-2)=-(x-4)$

$$3y+6=4-x\Rightarrow \boxed{x+3y=-2}$$

Topic H: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

1.) $x^2 + x + \frac{1}{4} = 0$

$\left(x + \frac{1}{2}\right)^2 = 0$

$x = -\frac{1}{2}$

2.) $2x^2 - 72 = 0$

$2\left(x^2 - 36\right) = 0$

$(x+6)(x-6) = 0$

$x = -6 \text{ or } 6$

3.) $81x^2 + 72x + 16 = 0$

$(9x+4)^2 = 0$

$x = -\frac{4}{9}$

4.) $x + \frac{1}{x} = \frac{17}{4}$

$x^2 + 1 = \frac{17}{4}x, x \neq 0$

$4x^2 + 4 = 17x$

$4x^2 - 17x + 4 = 0$

$(4x-1)(x-4) = 0$

$x = \frac{1}{4} \text{ or } 4$

5.) $x^3 - 5x^2 + 5x - 25 = 0$

$x^2(x-5) + 5(x-5) = 0$

$(x-5)(x^2+5) = 0$

$x = 5$

Topic I: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) $y=\dfrac{2x^2+6x}{x^2+5x+6}$

$y=\dfrac{2x(x+3)}{(x+3)(x+2)}$

Hole : $x=-3$

$y=\dfrac{2x}{x+2}\Rightarrow y(-3)=\dfrac{-6}{-1}=6$

Vertical Asymptote : $x=-2$

$y\to$ infinite form

$\dfrac{2(-2)(-2+3)}{(-2+3)(-2+2)}=\dfrac{-4}{0}\to\infty$

Horizontal Asymptote : $y=2$

as $|x|$ gets large $y\to\dfrac{2x^2}{x^2}=2$

2.) $y=\dfrac{x}{x^2-25}$

$y=\dfrac{x}{(x+5)(x-5)}$

Vertical Asymptote : $x=\pm5$

$y\to$ infinite form $\dfrac{\pm5}{0}\to\infty$

Horizontal Asymptote : $y=0$

as $|x|$ gets large $y\to\dfrac{x}{x^2}=\dfrac{1}{x}=\dfrac{1}{\infty}=0$

3.) $y=\dfrac{x^3}{x^2+4}$

no Vertical Asymptote :

$x^2+4\neq0$

no Horizontal Asymptote :

as $|x|$ gets large $y\to\dfrac{x^3}{x^2}=\dfrac{x}{1}\to\infty$

4.) $y=\dfrac{x^3+4x}{x^3-2x^2+4x-8}$

$y=\dfrac{x(x^2+4)}{x^2(x-2)+4(x-2)}$

$=\dfrac{x(x^2+4)}{(x^2+4)(x-2)}$

Vertical Asymptote : $x=2$

$y\to$ infinite form $\dfrac{2}{0}\to\infty$

Horizontal Asymptote : $y=1$

as $|x|$ gets large $y\to\dfrac{x^3}{x^3}=1$

5.) $y = \dfrac{10x+20}{x^3 - 2x^2 - 4x + 8}$

$$y = \frac{10(x+2)}{x^2(x-2) - 4(x-2)}$$

$$= \frac{10(x+2)}{(x^2-4)(x-2)}$$

$$= \frac{10(x+2)}{(x-2)(x+2)(x-2)}$$

Hole : $x = -2$

$$y(-2) = \frac{10}{(-2-2)(-2-2)} = \frac{10}{16}$$

Vertical Asymptote : $x = 2$

$y \rightarrow$ infinite form $\dfrac{10}{0} \rightarrow \infty$

Horizontal Asymptote : $y = 0$

as $|x|$ gets large $y \rightarrow \dfrac{10x}{x^3} = \dfrac{10}{x^2} = \dfrac{10}{\infty} \rightarrow 0$

Topic J: Negative and Fractional Exponents

Simplify and write with positive exponents.

1.) -12^2x^{-5}

$= \frac{-144}{x^5} = \boxed{-\frac{144}{x^5}}$

2.) $(4x^{-1})^{-1}$

$= (4^{-1})(x^1) = \boxed{\frac{x}{4}}$

3.) $\left(\frac{5x^3}{y^2}\right)^{-3}$

$= (5)^{-3}(x^3)^{-3}(y^{-2})^{-3}$

$= \frac{y^6}{5^3x^9} = \boxed{\frac{y^6}{125x^9}}$

4.) $(x^3-1)^{-2}$

$= \boxed{\frac{1}{(x^3-1)^2}}$

5.) $(-32x^{-5})^{-3/5}$

$= (-32)^{-3/5}x^3 = \frac{x^3}{(-32)^{3/5}}$

$= \frac{x^3}{(-2)^3} = \boxed{-\frac{x^3}{8}}$

6.) $\frac{1}{4}(16x^2)^{-3/4}(32x)$

$= \frac{8x}{(16x^2)^{3/4}} = \frac{8x}{(16)^{3/4}x^{3/2}}$

$= \frac{8x}{(2)^3x^{3/2}} = \frac{x}{x^{3/2}} = \boxed{\frac{1}{x^{1/2}}}$

7.) $\frac{(x^2-1)^{-1/2}}{(x^2+1)^{1/2}}$

$= \frac{1}{(x^2-1)^{1/2}(x^2+1)^{1/2}}$

$= \boxed{\frac{1}{[(x^2-1)(x^2+1)]^{1/2}}}$

8.) $(x^{-2}+2^{-2})^{-1}$

$= \frac{1}{\left(\frac{1}{x^2}+\frac{1}{4}\right)} \cdot \frac{4x^2}{4x^2}$

$= \boxed{\frac{4x^2}{(4+x^2)}}$

Solutions

Topic K: Complex Fractions

complex fractions:

1.) $\dfrac{x^{-1}+y^{-1}}{x+y}$

$$=\frac{\frac{1}{x}+\frac{1}{y}}{x+y}\cdot\frac{xy}{xy}$$

$$=\frac{y+x}{(x+y)(xy)}=\boxed{\frac{1}{xy}}$$

2.) $\dfrac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

$$=\frac{\frac{1}{x^2}+\frac{1}{x}+1}{\frac{1}{x^2}-x}\cdot\frac{x^2}{x^2}=\frac{1+x+x^2}{1-x^3}$$

$$=-\frac{1+x+x^2}{x^3-1}=-\frac{1+x+x^2}{(x-1)(x^2+x+1)}$$

$$=-\frac{1}{x-1}=\boxed{\frac{1}{1-x}}$$

$$\begin{array}{r|l} & x^2+x+1 \\ x-1 & x^3 \qquad\qquad -1 \\ & \underline{x^3-x^2} \\ & x^2 \\ & \underline{x^2-x} \\ & x-1 \\ & \underline{x-1} \\ & 0 \end{array}$$

3.) $\dfrac{\frac{1}{3}(3x-4)^{-3/4}}{-\frac{3}{4}}$

$$=\frac{\frac{1}{3}(3x-4)^{-3/4}}{-\frac{3}{4}}\cdot\frac{12}{12}$$

$$=\frac{4}{-9(3x-4)^{3/4}}$$

$$=\boxed{-\frac{4}{9(3x-4)^{3/4}}}$$

4.) $\dfrac{2x(2x-1)^{1/2}-2x^2(2x-1)^{-1/2}}{(2x-1)}$

$$=\frac{2x(2x-1)^{1/2}-\frac{2x^2}{(2x-1)^{1/2}}}{(2x-1)}\cdot\frac{(2x-1)^{1/2}}{(2x-1)^{1/2}}$$

$$=\frac{2x(2x-1)-2x^2}{(2x-1)(2x-1)^{1/2}}=\frac{4x^2-2x-2x^2}{(2x-1)^{3/2}}$$

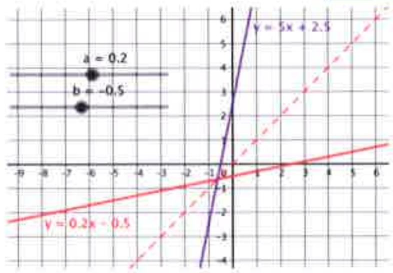
$$=\frac{2x^2-2x}{(2x-1)^{3/2}}=\boxed{\frac{(2x)(x-1)}{(2x-1)^{3/2}}}$$

Topic L: Inverses

Find the inverse of each of the following functions and use a graphing utility (TI-Nspire or Desmos) to show graphically that its inverse is a function.

1.) $y = ax + b$

Inverse : $x = ay + b \Rightarrow$ $\boxed{y = \dfrac{x-b}{a}}$

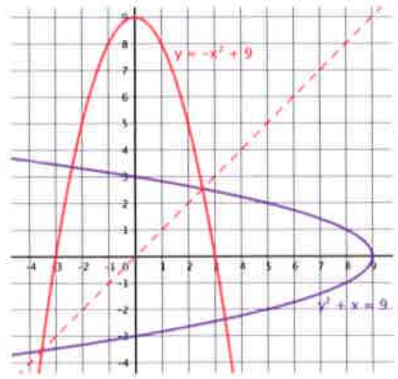


2.) $y = 9 - x^2,\ x \geq 0$

Inverse : $x = 9 - y^2$

$y^2 = 9 - x \Rightarrow$ $\boxed{y = \pm\sqrt{9-x}}$

inverse not a function

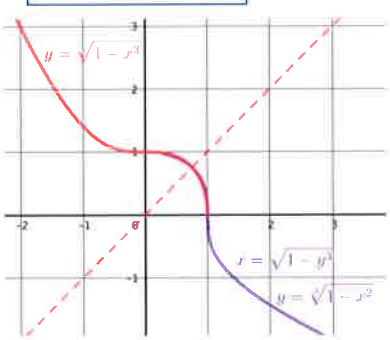


3.) $y = \sqrt{1 - x^3}$

Inverse : $x = \sqrt{1 - y^3}$

$x^2 = 1 - y^3 \Rightarrow y^3 = 1 - x^2$

$\boxed{y = \left(1 - x^2\right)^{1/3}}$



4.) $y = \dfrac{2x+1}{3-2x}$

Inverse : $x = \dfrac{2y+1}{3-2y}$

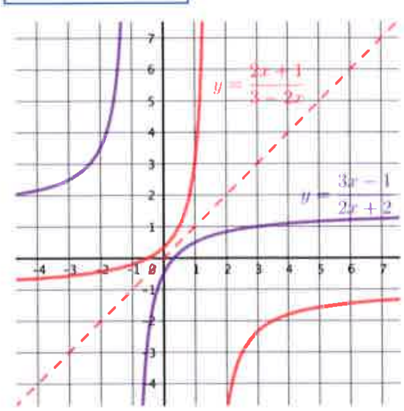
$x(3-2y) = 2y+1$

$3x - 2xy = 2y + 1$

$3x - 1 = 2y + 2xy$

$3x - 1 = (2 + 2x)y$

$\boxed{y = \dfrac{3x-1}{2x+2}}$



Find the inverse of each of the following functions and show that $f\left(f^{-1}(x)\right)=x$

5.) $f(x)=\frac{1}{2}x-\frac{4}{5}$

Inverse : $x=\frac{1}{2}y-\frac{4}{5}$

$\frac{1}{2}y=x+\frac{4}{5}$

$y=\boxed{2x+\frac{8}{5}=f^{-1}(x)}$

$f\left(f^{-1}(x)\right)=f\left(2x+\frac{8}{5}\right)$

$=\frac{1}{2}\left(2x+\frac{8}{5}\right)-\frac{4}{5}$

$=x+\frac{4}{5}-\frac{4}{5}=x$

6.) $f(x)=\frac{x^2}{x^2+1}$

Inverse : $x=\frac{y^2}{y^2+1}$

$x\left(y^2+1\right)=y^2$

$xy^2+x=y^2$

$y^2-xy^2=x$

$y^2(1-x)=x$

$y=\pm\sqrt{\frac{x}{1-x}}$ not a function

$$\boxed{f^{-1}(x)=\begin{cases}\sqrt{\frac{x}{1-x}}\\ \text{or}\\ -\sqrt{\frac{x}{1-x}}\end{cases}}$$

$f\left(f^{-1}(x)\right)=f\left(\pm\sqrt{\frac{x}{1-x}}\right)$

$=\frac{\left(\pm\sqrt{x/(1-x)}\right)^2}{\left(\pm\sqrt{x/(1-x)}\right)^2+1}$

$=\frac{\left(x/(1-x)\right)}{\left(x/(1-x)\right)+1}\cdot\frac{(1-x)}{(1-x)}$

$=\frac{x}{x+(1-x)}=\frac{x}{1}=x$

10.) Without finding the inverse, find the domain and range of the inverse to $f(x)=\frac{\sqrt{x+1}}{x^2}$

$\boxed{\text{Range: } f(x)\geq 0}$ where $f(-1)=0$ and there is a vertical asymptote at $x=0\Rightarrow y\rightarrow$ infinite form $\frac{+\#}{+0}\rightarrow+\infty$

$\boxed{\text{Domain of } f(x): x\neq 0 \text{ and } x\geq -1}$

$\boxed{\text{Range of } f^{-1}(x): y\neq 0 \text{ and } y\geq -1}$

$\boxed{\text{Domain of } f^{-1}(x): x\geq 0}$

Topic M: Adding Fractions and Solving Rational Equations

1.) Combine the following fractions:

a.) $\dfrac{5}{2x}-\dfrac{5}{3x+15}$

$$=\frac{5}{2x}\cdot\frac{3x+5}{3x+5}-\frac{5}{3x+5}\cdot\frac{2x}{2x}$$

$$=\frac{5(3x+5)-10x}{2x(3x+5)}=\frac{15x+25-10x}{2x(3x+5)}$$

$$=\boxed{\frac{5x+25}{2x(3x+5)}}$$

b.) $\dfrac{2x-1}{x-1}-\dfrac{3x}{2x+1}$

$$=\frac{2x-1}{x-1}\cdot\frac{2x+1}{2x+1}-\frac{3x}{2x+1}\cdot\frac{x-1}{x-1}$$

$$=\frac{(2x-1)(2x+1)-3x(x-1)}{(x-1)(2x+1)}$$

$$=\frac{(4x^2-1)-3x^2+3x}{(x-1)(2x+1)}=\frac{x^2+3x-1}{(x-1)(2x+1)}$$

$$=\boxed{\frac{x^2+3x-1}{(x-1)(2x+1)}}$$

2.) Solve the equation for *x*.

a) $\dfrac{5}{2x}-\dfrac{5}{3(x+5)}=\dfrac{5}{x}$

$$\left(\frac{5}{2x}-\frac{5}{3(x+5)}\right)(6x)(x+5)$$

$$=\left(\frac{5}{x}\right)(6x)(x+5)$$

$$15(x+5)-10=30(x+5)$$

$$15x+65=30x+150$$

$$-15x=85\Rightarrow\boxed{x=-\frac{17}{3}}$$

b.) $\dfrac{2x-1}{x-1}-\dfrac{3x}{2x+1}=\dfrac{x^2+11}{2x^2-x-1}$

$$\left(\frac{2x-1}{x-1}-\frac{3x}{2x+1}\right)(x-1)(2x+1)$$

$$=\left(\frac{x^2+11}{(x-1)(2x+1)}\right)(x-1)(2x+1)$$

$$(2x-1)(2x+1)-3x(x-1)=x^2+11$$

$$4x^2-1-3x^2+3x=x^2+11$$

$$x^2+3x-1=x^2+11\Rightarrow 3x-1=11$$

$$3x=12\Rightarrow\boxed{x=4}$$

Topic N: Absolute Value Equations

Solve the following equations:

1.) $|4x-5|+5x+2=0$

$(4x-5)\geq 0 \Rightarrow (4x-5)+5x+2=0$

$9x-3=0 \Rightarrow 9x=3 \Rightarrow \cancel{x=\frac{3}{9}=\frac{1}{3}}$

because $\left(4\left(\frac{1}{3}\right)-5\right)<0$

$(4x-5)\leq 0 \Rightarrow -(4x-5)+5x+2=0$

$-4x+5+5x+2=0 \Rightarrow x+7=0 \Rightarrow x=-7$

$\boxed{x=-7}$

2.) $|x^2-2x-1|=7$

$(x^2-2x-1)\geq 0 \Rightarrow$

$(x^2-2x-1)=7$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$\boxed{x=4 \text{ or } x=-2}$

$(x^2-2x-1)\leq 0 \Rightarrow$

$-(x^2-2x-1)=7$

$(x^2-2x-1)=-7$

$x^2-2x+7=0$

$x=\frac{2\pm\sqrt{4-4(7)}}{2}$ not real

Topic O: Solving Inequalities

Solve the following inequalities:

1.) $\frac{3}{4} > x+1 > \frac{1}{2}$

$\frac{3}{4} - 1 > x > \frac{1}{2} - 1$

$\boxed{-\frac{1}{4} > x > -\frac{1}{2}}$

2.) $x+7 \geq |5-3x|$

$(5-3x) \geq 0 \Rightarrow x+7 \geq 5-3x$

$4x \geq -2 \Rightarrow x \geq -\frac{2}{4} = -\frac{1}{2}$

$(5-3x) \leq 0 \Rightarrow x+7 \geq -(5-3x)$

$x+7 \geq -5+3x \Rightarrow -2x \geq -12 \Rightarrow x \leq 6$

$\boxed{-\frac{1}{2} \leq x \leq 6}$

3.) $(x+2)^2 < 25$

$-5 \leq x+2 \leq 5$

$\boxed{-7 \leq x \leq 3}$

4.) $\frac{5}{x-6} \geq \frac{1}{x+2}$

$\frac{5}{x-6} - \frac{1}{x+2} \geq 0$

$\left(\frac{5}{x-6} - \frac{1}{x+2}\right)\frac{(x-6)(x+2)}{(x-6)(x+2)} \geq 0$

$\frac{5(x+2)-(x-6)}{(x-6)(x+2)} \geq 0$

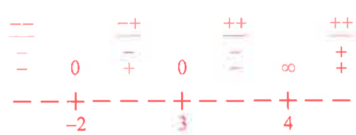
$\frac{4x+16}{(x-6)(x+2)} \geq 0$

$\boxed{-4 \leq x < -2 \text{ or } x > 6}$

5.) Find the domain of: $\sqrt{\frac{x^2-x-6}{x-4}}$

$\frac{x^2-x-6}{x-4} \geq 0$

$\frac{(x-3)(x+2)}{x-4} \geq 0$



$\boxed{-2 \leq x \leq 3 \text{ or } x > 4}$

Topic P: Exponential Functions and Logarithms

Simplify the following:

1.) $\log_2 \frac{1}{4}$

$\frac{1}{4} = 2^{-2}$

$\boxed{\log_2 \frac{1}{4} = -2}$

2.) $\ln \frac{1}{\sqrt[3]{e^2}}$

$\frac{1}{\sqrt[3]{e^2}} = e^{-2/3}$

$\boxed{\ln \frac{1}{\sqrt[3]{e^2}} = -\frac{2}{3}}$

3.) $5^{\log_5 40}$

$\boxed{5^{\log_5 40} = 40}$

4.) $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

$= \log_{12}(2 \cdot 9 \cdot 8) = \log_{12}(144)$

$\boxed{= \log_{12}(12^2) = 2}$

5.) $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$

$= \log_{\frac{1}{3}}\left(\frac{4}{3} \cdot \frac{1}{12}\right) = \log_{\frac{1}{3}}\left(\frac{1}{9}\right)$

$\boxed{= \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^2 = 2}$

6.) $\log_3\left(\sqrt{3}\right)^5$

$= 5\log_3\left(3^{1/2}\right)$

$\boxed{= 5\left(\frac{1}{2}\right) = \frac{5}{2}}$

Solve the following:

7.) $\log_9\left(x^2 - x + 3\right) = \frac{1}{2}$

$x^2 - x + 3 = 9^{1/2}$

$x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\boxed{x = 0 \text{ or } x = 1}$

8.) $\log(x-3) + \log 5 = 2$

$5(x-3) = 10^2$

$5x - 15 = 100 \Rightarrow 5x = 115$

$\boxed{x = \frac{115}{5} = 23}$

9.) $\log_5(x+3) - \log_5 x = 2$

$\frac{x+3}{x} = 5^2 \Rightarrow x + 3 = 25x$

$-24x = -3$

$\boxed{x = \frac{3}{24} = \frac{1}{8}}$

10.) $3^{x-2} = 18$

$\frac{3^x}{3^2} = 2 \cdot 3^2 \Rightarrow 3^x = 2 \cdot 3^4$

$\boxed{x = \log_3 2 + 4 \approx 4.6309\ldots}$

11.) $e^{3x+1} = 10$

$3x + 1 = \ln 10$

$\boxed{x = \frac{\ln 10 - 1}{3} \approx 0.4341\ldots}$

12.) $8^x = 5^{2x-1}$

$x \ln 8 = (2x-1)\ln 5$

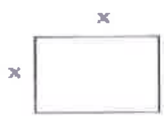
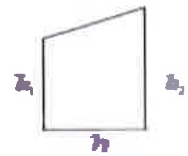
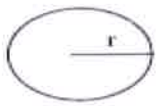
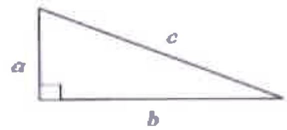
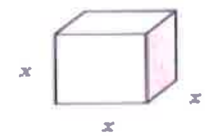
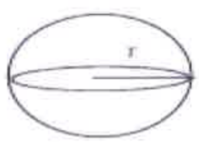
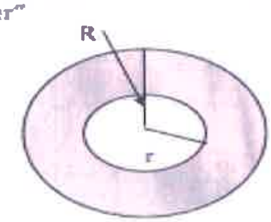
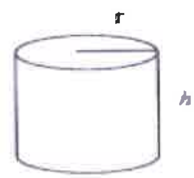
$x \ln 8 = (2\ln 5)x - \ln 5$

$(\ln 8 - 2\ln 5)x = -\ln 5$

$\boxed{x = -\frac{\ln 5}{\ln 8 - 2\ln 5} \approx 1.4124\ldots}$

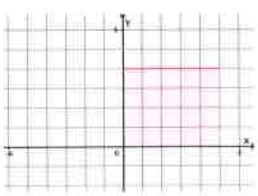
Topic Q: Geometry

1.) You will use each of the following formulas in AP Calculus AB. Complete each of the following.

Square  Perimeter = $4x$ Area = x^2	**Rectangle**  Perimeter = $2x+2y$ Area = xy	**Trapezoid**  Area = $\frac{1}{2}(b_1+b_2)h$
Circle  Circumference = $2\pi r$ Area = πr^2	**Triangle**  Pythagorean Theorem (only good for right triangles) = $a^2+b^2=c^2$ Area (of any triangle) = $\frac{bh}{2}$	**Cube**  Volume = x^3 Surface Area = $6x^2$
Sphere  Volume = $\frac{4}{3}\pi r^3$	**"Washer"**  Area of the shaded region = $\pi R^2 - \pi r^2$	**Cylinder**  Volume = $\pi r^2 h$

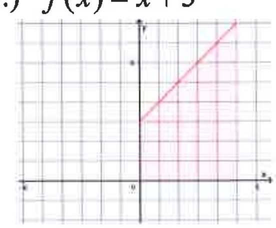
Find the area between the x-axis and $f(x)$ from $x = 0$ to $x = 5$. Sketch the region to verify.

2.) $f(x) = 4$



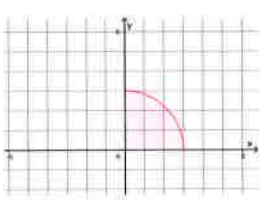
$area = (4)(5) = 20$

3.) $f(x) = x + 3$



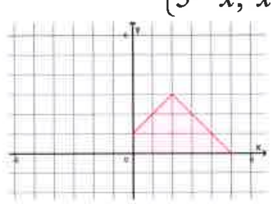
$area = \frac{1}{2}(8+3)(5) = 27.5$

4.) $f(x) = \sqrt{9 - x^2}$



$area = \frac{1}{4}\pi(3)^2 = \frac{9\pi}{4} = 7.0685$

5.) $f(x) = \begin{cases} x+1, & x \le 2 \\ 5-x, & x > 2 \end{cases}$



$area = \frac{1}{2}(1+3)(2) + \frac{1}{2}(3)(3) = 8.5$